THE HIGHER NUMERALS IN EARLY NORDIC TEXTS, AND THE DUODECIMAL SYSTEM OF CALCULATION.

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A central theme for understanding Early Nordic and Germanic civilization is to evaluate the ability of calculation - as the lack of such skills sets serious limitations to the development of the society. It is well known that the decimal system of calculation came to Europe late in the Middle Ages, and besides the foreign Roman numerals, what kind of arithmetic calculation of their own did these people employ?

Anglo-American scholars do not hesitate when describing the Germanic peoples that took over Europe from the Romans in the 5th century as innumerate as well as illiterate: They possessed neither elaborate systems of numbers, weights, and measures, nor sophisticated techniques for calculating with these quantities and recording the results.

This information is however at odds with the archaeological and linguistic evidence. Rulers have been found dating back to the Bronze age, and a measuring system in the "Trelleborg" fortresses has been found (2). The runic script might date back to the 3rd c. BC. On rune stones are mentioned the numbers: 2, 3, 4, 5, 8, 9, 12, 13, 20, 30, 90, hundred, (thousand 13 c.). (3).

The comparative philologists have studied the numerals intensely. They have discovered that the numerals from one to hundred have a common Indo-European root, and a ten base counting system is being used. However, some peculiar features can be observed: The ten count is crossed by a twelve count, a twenty count, a sixty count, and a hundred and twenty count has been found in the Germanic languages. (4).

At present particular attention will be drawn to the problem of the duodecimal system of calculation, i.e. the Germanic counting of hundreds being hundred and twenty, which has also been called the long hundred. The main problem is to reveal the inner logic of the system, and how such a system might function, and to establish if the Nordic "long hundred" is connected to more ancient systems of calculation. At this early stage of research it is only possible to draw some general and preliminary conclusions.

The philological study of the "long hundred" has a long history. It was already noticed by Jacob Grimm in 1819. About hundred years ago there was a considerable attention drawn to the long hundred. W. H. Stevenson thought that the long hundred was the original hundred of the Teutonic tribes, and he wrote that the long hundred is of very great importance, for it throws considerable light upon that early history of European nations that is being gradually unfolded by the study of comparative philology (5).

In his famous treatise, Johannes Schmidt stated that the incision between 60 and 70, as well as the "long hundred" was a proof of an influence from the Babylonian sexagesimal count, and accordingly, the home of the Indo-Europeans should be found in Asia (6). This article actually led the debate astray: Instead of describing the counting system, the debate became pro or con the Babylonian theory, until F. Sommer proved that there was no sign of any Babylonian influence (1950). Since then Rosenfeld, Frings, and Szemerényi have described the numerical system, but still the problem of the "long hundred" has been treated almost as a nonexistent phenomenon (7).

More recently the debate of the Germanic decades has been reopened by G. Schmidt, R. Lühr, and C.F. Justus, but without greater succes (8).

THE GERMANIC NUMBERS ELEVEN, TWELVE, THIRTEEN

Gothic	Old Norse	Old English	Old Saxon	Old High German
ainlif	ellifu	anleofan	elleban	einlif
twalif	tolf	twelf	twelif	zwalif
*þreis-taihun	þrettān	þri-tene	thri-tehan	drizehan

THE GERMANIC DECADES: 50, 60, 70, 80, 90, 100, 110, 120.

Gothic	Old Norse	Old English	Old Saxon	Old High German
fimf tigjus	fimm tøgr	fiftig	fif-tig	finf-zug
saíhs tigjus	sex tøgr	sixtig	*sehstig	sehs-zug
sibun-te-hund	sjau røðr sjau tøgr	hund-seofontig	antsibunta	sibun-zo
ahtau-te-hund	ātt røðr ātta tøgr	hund-eahtatig	antahtoda	ahto-zo
niun-te-hund	ní røðr ní tøgr	hund-nigon-tig	antnigonda	*niun-zo
taihun-te-hund taihun-taihund	tíróðr tíu tøgr hundrað tirøtt	hund-teontig	hund	zehan-zo
?	ellifu tigir	hund-endleofan-tig	?	?
hunda (?)	hundrað (tolfrøtt)	hund(twelftig)	Chunn-tualepti (Lex Salica)	?

The break in the sequence of number words between 12/13, and 60/70 can easily be observed. "120" might have disappeared in some Germanic languages, as it is mainly attested in Old English and Old Norse. This led Frings to consider that the "long hundred" should mainly be considered to be a Baltic/North Sea phenomenon. (The Old Norse -ræðr is supposed to be older than the -tigir, which might be an Icelandic innovation). - It might be concluded that in the Germanic languages two forms of hundred can be found: A ten-counted hundred (100), and a twelve-counted hundred (120).

However, the comparative philological method has serious limitations, as mainly phonological implications are considered, whereas arithmetical considerations are not treated. In order to understand the "long hundred" as a system, it is necessary to introduce some facts about ancient arithmetic calculation.

Ancient arithmetic systems differ substantially from our modern decimal calculation. The series of numbers has no definite beginning (no zero) but has definite limits (e.g. like the Roman numerals X, L, C, D, M), as stated in a Medieval text: 'The periodical progression of numbers or some units, seem to grow through some finite forms; the first progression of numbers is from one till ten. Second from ten till hundred. Third from the hundred number till the thousand (9). The modern decimal counting system has a definite beginning (zero), but no definite limits (10, 100, 1000 etc. are not limits, but rather new beginnings of number series).

The order of the factors is not unimportant: 12 x 10 is not the same as 10 x 12, as the calculation is performed by either 10 in the former example or by 12 in the latter.

A fundamental principle in ancient calculation is to pay attention to the divisors, as fractions pose a major problem. It is especially important that unit fractions become integers: 10 has only 2 and 5 as divisors, whereas 12 has 2, 3 4, and 6 - so 12 is more practical for calculation.

The almost universal adoption of the decimal calculation is undoubtedly due to the fact that we have ten fingers, but although being anatomically convenient it has only few advantages from an arithmetical viewpoint, having only 2 and 5 as divisors. An alternative is the vigesimal system, which appears in the French, Danish, Albanian, Celtic, and Basque languages. In the counting by scores 2, 4, and 5 are divisors, but still thirds are incompatible with the system.

The popularity of the duodecimal calculation is probably due to the many divisors (2, 3, 4, 6), but twelve could also be counted on the fingers: On one hand the four fingers have a total of 12 joints, and a duodecimal finger counting based on counting the joints, combined with counting 5 fingers on the other hand (5 x 12) is still widely used in Asia (10).

The higher powers of the base can be constructed in several ways. A regular system would be to use powers of the base, e.g. 10 x 10 or 12 x 12. But historical evidence clearly demonstrate that hybrid constructions were often used, e.g. the sexagesimal system of the Babylonias (6 x 10), and the "long hundred" (12 x 10). The reason for constructing hybrid systems of calculation is probably to combine the advantages of both the decimal, and the duodecimal systems, as 2, 3, 4, 5, 6, (8), 10, 12, and 20 are basic divisors.

It is very difficult to reveal the construction of the "long hundred" counting system. The proof can only be found in arithmetic problems, or numbers where the exact result is known, but such information is rarely found in Medieval literature. The following examples reveal, how addition and multiplication were performed:

They had hundred men, and they went to Jomsborg - 80 men were sworn, 40 went away (11).

The 365 days of the year are explained as three hundred and five days (12).

The 532 years of the easter cycle are: four hundred and 40 + 12 (13).

King Olaf the Holy came with four hundred men from the Swedish king; his brother met him with six hundred men; a total of twelve hundred men is mentioned (14). - This only makes sense if the small hundreds are long hundreds, whereas the "thousand" consist of twelve "short hundreds", i.e. 480 + 720= 1,200.

Arent Berntsen explains the counting of fish on Iceland: 1 load of fish is 1,000 fish. "100" fish are 6 score or= 120 fish. 3 commercial "væt" of fish are "100" fish; as 1 væt is 40 fish, 3 væt is 3 x 40= 120 fish. When "100" is 120, then "1,000" must be 1,200 even if this is not explained (15).

The following examples show quite a different construction, the division:

It was reported that six navies had been seen each with one king, every of them contained five thousand ships, and on each was three hundred men. Even so it was indicated that every thousand of the total was held together in four wings, and with the thousand was understood thousand and two hundred, as each wing comprised of three hundred men in number (16).

Bede (731) has a similar example: Wight has twelve hundred hides, the bishop was given three hundred hides, which was a quarter of the total (17).

Arent Berntsen explains that in Jutland the fish whiting is counted: 15 score is one Quarter; 4 Quarter is "1,000" = 1,200. (18).

These examples give the following equations: One thousand= 1,200 : 4= 300, or 4 x 300= 1,200. The thousand mentione is a "long thousand", but it is divided into "small hundreds" of "100", which obviously seem to contradict the extensive evidence of a "hundred" being "120", and also its multiples are commonly found in Medieval texts. But from the examples it can be seen that the long and the short hundred interact. In Old Norse the two kinds of "hundred" is often indicated as "hundrað tirøtt" or "hundrað tolfrøtt", but often the hundred mentioned is unqualified so that it is impossible to observe, which hundred was meant, except maybe from looking at the context (19).

EXAMPLE 1.

ROUND NUMBERS IN THE 120 COUNTING SYSTEM:

MULTIPLICATION ENTER: | DIVISION ENTER:

1	2	3	4	5	6	7	8	9	10	[11]	[12]
12	24	36	48	60	72	84	96	108	120		
10	20	30	40	50	60	70	80	90	100	110	120
120	240	360	480	600	720	840	960	1080	1200		
100	200	300	400	500	600	(700)	800	900	1000	(1100)	1200
1200	2400	3600	4800	6000	7200	8400	9600	10800	12.000		
1000	2000	3000	4000	5000	6000	(7000)	8000	9000	10.000	(11.000)	12.000
12.000	24.000	36.000	48.000	60.000	72.000	84.000	96.000	108.000	120.000		
10.000	20.000	30.000	40.000	50.000	60.000	(70.000)	80.000	90.000	100.000	(110.000)	120.000
1/12	(1/8)	1/4	1/3		1/2		2/3	3/4			1 FRACTION

A theoretical outline of the long hundred system.

The upper lines are for multiplication and addition, starting at the low end of the number series.

The lower lines are for division, starting at the top end of the series, the quarter and the half is underlined.

The point of entering into each system is double underlined, and shown with a pointer in the margin.

LONG HUNDREDS (of 120) AND NARROW HUNDREDS (of 100) IN ICELANDIC SAGAS.

(compare with example , the hundred lines).

ONE HUNDRED	TWO HUNDRED	THREE HUNDRED	FOUR HUNDRED	FIVE HUNDRED
120 (1x120)	240 (2x120)	360 (3x120)	480 (4x120)	600 (5x120)
halfur tolfta teugur (Rim 2,95) hundrad tolfrøtt (Rim 2,77) tolfreott hundraþ (Flat,I 271-16.Homil. 23-3. DN I,344-12)	halft annat hundrad (Rim 2,95+156) tvav hundroð tolfræð(xIIR) (Jb 151-2. GrgSt.352-12) CC tolfræð (XII rød) (Jb 162-12. OH 4-21)	þriu hundrut tolfredh (Rim I,9, II,76-5,139 CCC tolfreoþ (Rim 1,12) CCC daga/natta (Rim 1,65+9	flogur hundrut tolf raed (Rim 2,157) CCCC tolfraed (Rim 2,173) fiorda hundraþs (Ari IV,1,6).	halft fimta hundrad (Dipl.Isl. 2,438) halft V hundrad (Dipl.Isl. 2,446)
100 (1x100)	200 (2x100)	300 (3x100)	400 (4x100)	500 (5x100)
Hundrad tirætt (Sk III,43.Fl.VIII,33 C.tirætt ok XX.Stj.55· V hins tiunda tigher (Rim 1,34) tiu tigu = C tirætt (Kristnisaga)	tvö hundrud tiræd sem C tvö i látinu (Grammar) halft annat hundrad (Hauksbok 151) II hundrud ok XL (SnE(W) 21-12)	þridia hundrads (Rim 1,64) CCC daga tiredum (Rim 1,61)		fimm hundrod oc xxx oc ii (Rim 1,62) fimm hundrut tiræð 32 ar (rim II,137-18) fimm hundrud + firum (Grm 23+24)

SIX HUNDRED	SEVEN HUNDRED	EIGHT HUNDRED	NINE HUNDRED	TEN HUNDRED
720 (6x120)	840 (7x120)	960 (8x120)	1080 (9x120)	1000/1200 (10x100/120)
	DCC tolfræd (pattr porv.)	átta hundraða tolfræd (DN II, 79-36)	halft niunda hundrad (Dipl.Isl. 1,204)	þusund (Sverre saga,Fm 8,448) X hundrud (StuK 213-18)
600 (6x100)	700 (7x100)	800 (8x100)	900 (9x100)	tio hundrodom (Hamd.22,Dpl.155-24. (Nj.338-9.StuR,15-11) tolf hundrod (Helgi Hund.I,25)
setta hundrad tireds (Rimb 61-14) setta hundrads tiroeds (Rim 1,32)	*siau hundrod manna (Gudr. III,7)	*atta hundrud (Grm 23) VIII hundrut tireð (Alex. 112-31)	hundrud niu (Hym. 8)	XII hundredum tyraedum (Rymb.p.404) XII hundrat tolfredh (Rim. 1,58)(mistake) tolf hundrud

The examples show that a twelve square construction is not the principle of construction of the "long hundred", and the "long thousand" should not be 12 x 120 (= 1,440) even if examples of such a construction are found (20).

From the mentioned examples it can be deducted that the "long hundred" counting system must be constructed according to a "formula" 12×10^n, which gives the higher bases 120, 1,200, 12,000, and 120,000. The system must consist of two parts: one system for multiplication, addition, and subtraction, and another for division, see example 1. In the upper lines of the system, multiplication and addition takes place from the low end of the number series (120 + 120= 240 etc., 120 x 7= 840). In the lower lines of the system, division takes place from the top end of the series (1,200 ÷ 4= 300. Both 300 and 360 might be called three hundred!).

Within the basic number series from 1 to 12, the two systems are almost identical. The double system appears first in the decades, but is used in an irregular way, as addition and multiplication should be performed in the upper system by calculating by twelves, and not by tens, as it is usually. The passage from ones to tens is therefore not consistent, which might be reflected in the irregular linguistic construction of the words "eleven" and "twelve". The number 120 should be considered as the last decade.

With the passage from tens to hundreds, the system becomes regular, and the system continues regularly into the thousands. It must be noticed, that I have only found the divisional mode used together with the half and the quarter of 1,200, but other fractions could be hidden behind the usage of the number hundrað-tirætt. It must also be noticed that the Roman numeral "C" centum has both the meaning "100" and "120" in Old Norse as well as in Old English, and even in latin texts. Numbers like 700, 7,000 etc. and 1,100 and 11,000 are less likely to find in the texts, as these numbers are not unit fractions of 12 ((i.e. 1,200 or 12,000).

Till now only round numbers have been mentioned, but how to reach other numbers is quite complicated to explain, as three different principles were employed: the under count, the over count, and the subtraction. These metods were used with both the long and the short hundred, with Roman numerals or spelled.

A usual method was to count to the round number under the number in question, just as we do today (e.g. 50 + 8= 58). This is probably the youngest method.

Then one could count on the basis of the round number over the number in question (e.g. 8 on the way to 60= 58). This method is still used in German and in Scandinavian fractions of smaller numbers, and probably it is more original.

Finally a subtraction method was used (2 from 60= 58). - This is extremely complicated, but some examples can make it more understandable:

The 364/365 days of the year: (Alfrædi: Rimtol).

þriu hundrut tolfredh ok V netr um fram (9-4). (+ 76-1).
= CCC natta ... um fram V netur (9-6).
CCC tõlfreðþ (12-13). (+ 57-20).
CCC daga oc IIIIa (65-1). (+ 156-1).
fiora daga ens fiorþa hundraþs (Ari, IV, 1.6).

It can easily be seen that in the first example, "three hundred" means 360. In the next example "C" means 120, so: CCC= 360. CCC can also be combined with the word tõlfreðþ, which means that CCC must be a translation of the linguistic expression "three hundred". These examples illustrate the "under count". The last example illustrate the "over count", in which the smaller units lie within the last mentioned hundred, i.e. four of the fourth hundred (i.e. 360 + 4 of the fourth "120" = 360 + 4).

The 532 years of the easter cycle: (Alfrædi: Rimtol).

I Pascha aulldh ero DXXXII ar (47-3).
V. C. XXX ok II år (200-6).
CCCC vetra tolfræd ok tveir vetur enns setta tigar (173-12).
fiogur hundrut vetra tolf ræd ok 40 vetra ok 12 vetur (157-11).
tveir hins fiorda tigar ens setta hundrads tireds (32-12).

The two first examples show regular Roman numerals. In the third example "C" is 120, so: 4x120= 480, and in "over count" two in the sixth ten= 52. (=532). The last example show the over count: two in the fourth ten (= 32) in the sixth hundred (= 532).

Relics of the overcount can be found in other Germanic languages, e.g. in German: anderthalb (1½). In Old English: feorðe healfund (half fourth hundred) and þridde healf hund (half of the third hundred, i.e. 250 or three long hundreds).

The overcount is very rarely found outside the Germanic territory. It has been borrowed into Finnish and Estonian, from the Germanic languages, and is found in Old Turkish numerals in the 8th century inscriptions in Mongolia (21). The roots of the Germanic overcount is therefore quite uncertain.

When turning to the "long hundred", several examples of its use can be found outside Scandinavia, showing its dissemination in space and time.

An evident example of a Scottish "long hundred" can be found in the Bute manuscript (14 c.), which has a marginal enumeration of the chapters throughout the volume, in which six scores go to the hundred (22).

In the English material, the Domesday Book mentions a "long hundred" as "Numero Anglice" (23). In the Anglo-Saxon Chronicle the Andredes forest is 30 x 120 miles, 120 men were killed in a battle at Wight, 120 ships perished, 840 men killed (i.e. 7 long hundreds) (24). There are several examples of "long hundreds" in Bede beside the one already mentioned, e.g. there are 300 hides to Man, 600 to Thanet, and 960 to Anglesey, and a grant of 120 hides (2 x 6 x 10) (25). In the tribal hidage, one of the earliest estimates of the English land, a series of hundreds and thousands can be observed. The hundreds are 300, 600, 900, and at the top end 12 hundreds (26). The multiples of thousands do not correspond to "one thousand" being 12 hundreds.

At present only a short sketch of the Celtic counting system can be given. In the Annals of Ulster, the basic unit is the score: 3 to 12 score are mentioned. The score and the hundred mentioned overlap (27). In Irish cēt means 100, but cēad can be "120" (6 x 20). In Cornish "cant byr" means "100", whereas "cant hir" is the "long hundred" (28).

At present the continental material will not be analyzed, but it may be noticed that the short hundred is mainly used. An exception is the "Lex Salica" in which fines are stipulated according to the duodecimal system: "unum tualepti sunt denarii CXX, culp. iud." (29).

When moving back in time into the Antiquity, an absolute ten count was predominating all over the Mediterranian world. This is known not only from the Roman numerals (I, V, X, L, C, D, M), but also from the Greek and Egyptian counting systems, which all have a ten base.

But in spite of the predominance of the decimal count, a duodecimal system can be found in practical calculation and metrology. E.g. the Roman army has 1,200 hastates, 1,200 principes, and 600 triarias (Polyb. 6,19 ff). A maniple consisted of 12 lines, 10 persons deep (= 120). The Julian Colonia Fano Basilica measures 120 x 60 feet (Vitruvius, v,i,6. See also the weight of water pipes v.2, and computing of ballistic missiles x,xi,3 etc.).

But most interesting is the Roman field measurement system. An ideal field called "Actus" is 120 square feet, but two of them are combined in a "Jugerum", which is 120 x 240 feet (Columella v,i,9 ff.).

The Jugerum is subdivided in duodecimal fractions. The method was probably to retain the length (240 feet), and to divide the breath in 12 units each of 10 feet. So the Roman field measurement system was obviously based on a combination of a twelve and a ten count.

It will lead too far to examine the Antique Roman and Greek weight and measurement systems, but it can be noticed that the duodecimal system prevails (often compatible with a formula: 12 x 12 x 10) (30).

The Mycenian culture dates to about 1500 B.C. and is one of the earliest evidence of Indo-European culture. Even if the decimal count is used solely for ordinary calculation, the duodecimal system is evident in weights and measures (e.g. 10 x 12 x12 (or 6)) (31).

As early as the end of the fourth millennium B.C., proto-Sumerian, and proto-Elamite scribes had well developed systems of numbers and measures, which included precursors of our own decimal system. Even if the system of calculation is sexagesimal (60 x 60 etc.), it must be noticed that it is compatible with the Greek and Mycenian weight and measurement systems (32).

In conclusion it could be said that in Scandinavian and Germanic societies it is possible to find that the number system had a peculiar hybrid construction, in which the word "hundred" had both the meaning "120" and "100", and also the "thousand" could be "1,200" or 1,000. A "mathematical" explanation would be a 12 x 10 square construction, and not a 10 x 12 square construction, which is only to be found in Antique Greek and Roman metrology. The "long hundred" might be explained as an influence from metrological systems on the linguistic system of calculation, but at present it is impossible to indicate, where and when such an infusion took place.

The Germanic overcount is an extremely unique and complicated way of expressing numbers, and an origin for the overcount is still to be suggested.

It can be seen that the Scandinavian and Germanic people had extremely elaborate systems of calculation of their own. Hence, they can not be considered to be neither innumerate nor illiterate. Our ancestors can therefore not be considered to belong to the category of "primitive culture" in a social anthropological context. At least the character of Old Norse civilization should be reconsidered.

NOTES.

1. M. S. Mahoney: Mathematics. In: Dictionary of the Middle Ages, vol. 8, p. 205. N.Y., 1987.
   A. Murray: Reason and Society in the Middle Ages. Oxford, 1978. pp. 143-4.
2. Helge Nielsen: MOSES - et PC-program til påvisning af "skjulte" måleenheder. In: Kark nyhedsbrev, 1990, Nr. 1.
3. S.B.F. Jansson: Runes in Sweden. Sth., 1987. e.g. p. 24, 32, 63, 66, 77, 121. KLNM, bd 11, sp. 421.
5. W.H. Stevenson: The Long Hundred and its Use in England. In: The Archaeological Review, vol. 4, no. 5. pp. 313-327. London, 1889.
   J. Grimm: Geschichte der deutsche Sprache, p. 238 ff. Urverwandtschaft.
4. A. Karker: Talsystem. KLNM, bd 18, sp. 118 ff.
6. J. Schmidt: Die Urheimath der Indogermanen und das europäische Zahlsystem. In: Abhandl. d. kön. Akad. d. Wiss. zu Berlin. 1890. p. 1 ff.
7. F. Sommer: Zum Zahlwort. In: Sitzungsber. d. Bayerischen Akad. d. Wissenschaften. Ph.-hist. Kl. Jahrg. 1950, hft. 7.
   H.-Fr. Rosenfeld: Die germanischen Zahlen von 70-90 und die Entwicklung des des Aufbaus der germanischen Zahlwörter. In Wissensch. Z. der Ernst Moritz Arndt-Universität Greifswald. Ges. u. Sprw. R. Nr. 3. Jahrg. 6 1956/7.

Szemerenyi, O.: Studies in the Indo-European system of numerals. 1960.
Th. Frings: Ingwäonisches in den Bezeichnungen der Zehnerzahlen. In: Beitr. z. Geschichte d. deutschen Sprache, 84. Halle, 1962. (before edited in 1960).
The following works fall outside the main debate:
O.S. Reuter: Zur Bedeutungsgeschichte des hundrad im Altwestnordischen. In: Arkiv för nordisk filologi. Vol. 49. Lund, 1933. pp. 36-67.
O.S. Reuter: Urnordischer und eurasischer Zählbrauch. In: Mannus, bd 25, h. 4. Leipzig, 1933. pp. 353-383.
K. Menninger: Zahlwort und Ziffer. Bd 1. Göttingen 1957.

8. G. Schmidt: Zum Problem der germanischen Dekadenbildungen. In: Zeitschr. f. vergleichende Sprachforschung. bd 84, 1970. p. 89 ff. Because Sommer has shown that the "long hundred" is not Babylonian, it is not analyzed. R. Lühr: Die Dekaden '70-120' im Germanischen. In: Münchener Studien zur Sprachwissenschaft, hft. 36. München, 1977. p. 59 ff.
C.F. Justus: Indo-European Numerals and Numeral Systems. In: Yoël Arbeitman: A Linguistic Happening in Memory of Ben Schwartz. Louvain-la-Neuve, 1988. p. 521 ff.
9. Alcuin, Epistolae, p. 133.
10. G. Ifrah: From One to Zero. N.Y. 1985. p. 65-66.
11. Flat. I, 172-37. 173-7. Jomsv. 66-23.33. Mork. 126-13 ff. Isl. 4 (7-23 ff).
12. Alfrædi Islenzk. II Rimtol. Udg. N. Beckman og Kr. Kålund. Kbh., 1914-16. p. 9, 12, 57, 65, 76, 143, 156, 175.
13. Alfrædi Islenzk, op. cit. p. 530.
14. Snorre Sturlason: Kongesagaer. overs. A. Holtsmark & D.A. Seip. 1975. p.441.
15. Arent Berntsen: Danmarckis oc Norgis Fructbar Herlighed. 1656. 1971. p.530.
16. Sex classium senos reges, earumque quamlibet quina navium milia complectentem vidisse se retulit, quarum unamquamque trecentorum remigum capacem esse constaret. Quamlibet vero tocius summe millenarium quaternis alis contineri dicebat, volebat autem millenarium mille ac ducentorum capacem intelligi, cum ala omnis trecentorum numero compleatur. Saxo: Gesta Danorum, book 5. Ed. Alfred Holder, 1886. p. 155.
17. Bede's Ecclesiastical History of the English People. Ed. B. Colgrave & R.A.B. Mynors. Oxford, 1972. p. 382-3.
18. Berntsen, op. cit. p. 552.
19. Reuter, Arkiv... op. cit. p. 36-37.
20. Alfrædi Islenzk, op. cit. p. CXXXVIII, 128, 58. KLNM, bd 7, sp. 84.
21. Reuter, Mannus, op. cit. p. 376 ff.
22. The Bute Manuscript. Acts of Scotland. p. vii.
23. D. B. i 336a col. 1 & 2.
24. D. Whitelock: The Anglo-Saxon Chronicle. The years c. 850-900.
25. Bede, op. cit. p. 72/3, 162/3.
26. W.G. Birch: Cartularium Saxonicum. Vol. 1. London, 1893. pp. 414-5. - See also F.W. Maitland: Domeday book and beyond. Cambridge, 1897. pp. 506-8.
27. Annals of Ulster. Ed. W.M. Hennessy. Vol. 1. 1887. Years: 836, 847-917.
28. H. Pedersen: Vergleichende Grammatik d. Keltischen Sprachen, bd. 2. Bedeutungslehre. Göttingen, 1909-13. p. 130. § 475.
29. Lex Salica. Incipiunt Chunnas.
30. H.-J. Alberti: Mass und Gewicht. Berlin, 1957. p. 34 ff. - See also my paper held at the Kalamazoo Conference, to be edited by Ronald Edward Zupko.
31. J. Chadwick: Linear B, and related scripts. 1987. p. 31-2.
32. J. Friberg: Numbers and measures in the earliest written records. Scientific American, feb. 1984. p. 81.