ARITHEMTIC IN SCANDINAVIAN AND SCOTTISH ACCOUNTING.

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Before Arab ciphers became common in Europe in the 15th-16th century, there existed no universal numerical system or uniform methods of calculation. Just as weight and measurement systems changed with the location, so did the arithmetic vary from place to place, according to local traditions and specific needs - and variations occurred over time. One of the most peculiar features in early European arithmetic was that two major systems of arithmetic existed: in southern Europe decimal calculation dominated (which should not be confused with modern place-value notation); in northern Europe the dominant decimal calculation was expanded by a duodecimal element [12] into a "long hundred" arithmetic, in which "hundred" had the numerical value "120". Roman numerals were used in both systems only to record numbers, since calculation were carried out on the counting-board, and the number system was designed for counting objects. Over time decimal calculation replaced the "long hundred" arithmetic, especially after Arabic numerals and written calculation had replaced Roman numerals and the counting-board.

The purpose of my paper is to demonstrate that "long hundred" was an arithmetical system of calculation, and to show how it functioned in comparison with decimal calculation; then I will present examples of "long hundred" calculation from Norwegian, Danish, Scottish, and Icelandic sources, and finally I would like to suggest a possible origin for the "long hundred."

The study of early arithmetic, and the "long hundred" in particular, is a potential growth area. The history of arithmetic is one of the most neglected and confused areas in medieval studies, in which scholars do not hesitate to draw far reaching conclusions on the basis of insufficient data. There exist no reliable introduction to medieval arithmetic and accounting except for Karl Menninger's forty year old study of the history of numerals and numbers.[1] The lack of text editions means that numerical data has to be searched by reading all extant medieval sources.

The linguistic study of the "long hundred" began with Rasmus Rask and Jacob Grimm in the early 19th century, and several German linguists have continued the study of the Germanic numeral system.[2] Contemporary scholars have ignored the numerical reality of the "long hundred" (that buying a "hundred" items involved payment for a quantity of 120 items). It is not as Keith Thomas sees it, one of the "styliser quantities like the baker's dozen, into which there entered an element of gift exchange or 'complimentary excess,' like the modern tradesman's discount."[3]

Even though examples of "long hundreds" are mainly found in sources from the British Isles and in Icelandic sagas, scholars have argued that its origin was Scandinavian and Germanic. The term "long hundred" was coined in 1889 by W.H. Stevenson, who thought that the it was introduced in the British Isles by the Normans.[4] Liebermann showed that the principle was firmly established in Anglo-Saxon law[5]. R. L. Poole erroneously called the long hundred calculation in England "duodecimal," and suggested a connection between the adoption of decimal numeration and the introduction of the abacus; but Charles Johnson thought it hard to believe that the introduction of counting by the score and the hundred was as recent as the 11th - 12th century.[6] Most economic histories barely notice the existence of long hundreds; for example, in his study of the English customs system, Gras observes that "the use of the great hundred of six score is several times illustrated."[7] O.S. Reuter carried out the most extensive study of the Icelandic "long hundred"but he had no knowledge of the long hundred in the British Isles.[8] I have examined the use of the "long hundred" in northern Europe in general.[9]

The Scottish long hundred has only recently attracted attention. In 1967 Peter Gouldesbrough warned that since the editors of the *Exchequer Rolls of Scotland* had translated the original figures from Roman numerals into Arabic equivalents, it would be easy to make mistakes, because the editors had not been aware that "in all numbers not relating to money the symbol 'c' represents the 'long' or 'great' hundred of six score."[10] He argued that the editors must have realized their mistake, when they translated the information 'vij^{c}v^{xx}vj' sheepskins, which became '700, 5 scoir, 6' in the edition, whereas the true Arabic equivalent is 946. Of great importance is Julian Goodare's article on the "long hundred" Scotland.[11] Professor Goodare argues that "the long hundred" was a Germanic method of reckoning, which reached Scotland from the south, against Alexis Easson, who claimed to have found a precursor to the long hundred in the seventh-century system of assessment in the kingdom of Dalriada, the *Senchus Fer nAlban*.[12] The economic historians Elizabeth Gemmill and Nicholas Mayhew have been interested in the "long hundred" so far as it interfered with the interpretation of the numerical information.[13] Historians have in general been unaware of the complexity of the linguistic interpretation of number words.[14]

The concept of "long hundreds" may seem confusing: it was not a "duodecimal" system (12^n) as one might expect, but a mixture of a ten and a twelve base. Medieval arithmetic was in general based on interlocking series of mixed-base systems, using mainly base ten in combination with other bases, of which twelve was the most common; in weight and measures the dyadic principle (counting by 2, 4, 8, 16 etc.) was also common. "Long hundred" calculation was frequently used in Icelandic and Scottish sources, where it has long been recognized that for non-monetary medieval and early modern numerical figures the word hundred (or "C") usually represents 120, instead of 100.

The "long hundred" must be regarded as a part an arithmetical system, because the decades had individual names up to 120 in old-English and old-Norse; it had an exact numerical value that was used in calculation, and finally it had a wide range of applications; for example, the Scots used it for calculating planks, cloth, fish, cattle, iron, nails, chapters in a book,[15] and the number of days of work. "The "long hundred" was an element in a system of "exponentiation" that also included a long thousand; where "C" meant "120", "M" meant 1,200 items. The wide range of applications must indicate that the "long hundred" was a general arithmetical system of calculation, and not merely a method for counting specific commodities.

It is important to realize that the medieval number concept is very different from ours. We think of numbers as points on a number line, but they thought numbers occupied the space between the points, and that they were spatial representations of the "volume" of the counted objects (like numbers on a keyboard). The number series was finite, and ran in series from one to ten, ten to one hundred, hundred to one thousand, etc. It consisted of unit fractions, sub-multiples, and integers. The concept of "zero" did not exist, except as the absence of objects. The rationale of "exponentiating" base 10 by using base 12 was to obtain as many divisors as possible, which would facilitate division. The "perfect number" 60 as well as 120 contain an especially large amount of divisors, which makes those numbers extremely easy to use for practical purposes, and therefore they are still used in the clock, the compass, electricity, photography, and in modern house construction.

The expression "hundred" normally meant 120 items, and the word "thousand" 1,200 items. But when counting years, money, and certain objects the words hundred and thousand had the traditional value. The double meaning of the words hundred and thousand was the result of a logical arithmetical procedure.

The decades continued from sixty, seventy, eighty, ninety, tenty, eleventy, to twelvety (the twelvety representing the hundred), and the hundreds continued to

twelve-hundred, which was a thousand. - Long hundreds were based on the principle of multiplying 10 by 12. It must, however, also be noted that the "long hundred" was often calculated by the score or " ^{XX} "; a long hundred would then contain six score, and a decimal hundred five score.

It is possible to establish how "long hundred" calculation functioned only by observing how it was used in practice, which shows that the "long" and a normal hundred interact. Base twelve was used once, while base ten was used repeatedly, which gives the following three possibilities for constructing a thousand:

(1) 12 x 10 x 10 - A normal hundred is combined with a "long" thousand
(2) 10 x 12 x 10 - A base ten "long hundred" and "long thousand."
(3) 10 x 10 x 12 - A base twelve "long hundred" and "long thousand."

Since the "long hundred" had two bases, it was also a double counting system: one system for multiplication, addition, and subtraction, and the other for division:

ROUND NUMBERS IN "LONG HUNDRED" CALCULATION

	1	2	3	4	5	6	7	8	9	10	11	12	
+	12	24	36	48	60	72	84	96	108	120			x
>	10	20	30	40	50	60	70	80	90	100	110	120	<
>	120	240	360	480	600	720	840	960	1080	1,200			x
x	100	200	300	400	500	600	700	800	900	1,000	1,100	1,200	<

The table explains theoretically how the "long hundred" functioned. In the basic number series, which goes from 1 to 10 or 12, the decades could be substituted by dozens. The "long hundred" appears in a regular form between 120 and a thousand of 1200; it is possible to add and multiply by 120: two hundred is "240," three hundred "360" and so on, up to ten hundred. The upper lines of the system present multiplication and addition starting from the lower end of the number series (120 + 120= 240 etc., 120 x 3= 360). When counting by the score, a "six score hundred" might represent multiplication and addition, whereas a "five score hundred" might represent division.

In the lower lines, division takes place from the top end of the series (1,200 ÷ 4= 300). In texts, both 300 and 360 may be called "three hundred" without indicating which hundred was meant. The most common division is by the half, quarter, three quarters, and the tenth part of 1,200. I have not presented any examples of this method of calculation in the present paper.

Evidence of the use of "long hundreds" is found from the earliest time until recently, but they are difficult to find because often nothing distinguishes them from normal hundreds. The "long hundred" does occur linguistically in old Norse and old-English, as an extension of the names of the decades from hundred to hundred-twenty. However, when they only used the word "hundred" or the Roman numeral "C" it is impossible to determine the numerical meaning intended. Only when an arithmetical procedure was involved can the numerical meaning of the expression "hundred" be determined, such as in additions of small sums, where the total becomes larger than hundred. or in Diophantine equations (X x Y= Z), where the value of a "hundred" items can be determined from the price paid:

"hundred" (or "C") items @ $ 1 an item= $ 120   .

Having presented some theoretical reflections on the construction of the "long hundred, I would like first to demonstrate how calculation was performed in Norway. Medieval tally sticks have been found at the excavations at the Brugge in Bergen, which show that people were numerate, but they do not show how they calculated. The earliest Norwegian account-books date from the sixteenth century, but the arithmetic was purely decimal and not based on "long hundreds." It might be a convention in Norwegian accounting not to use "long hundreds", which might be an influence from the intense international commerce with Dutch and German

merchants, who also used decimal calculation exclusively. Their accounting system may have been introduced from France at a much earlier date. The Norwegians used Roman numerals in a peculiar way; when a numeral is crossed by a line it is only half the value of the last unit. Where the calculation can be checked, it is decimal: (1 Marc= 16 ß)

j^c woger rottz to: Peither Høkæ, Hamburg, wogen for xxij ß, Summa j^{c}xxxviij– Marc[16]
  [100 Wog flounder @ 22 ß= 2,200 ß ÷ 16 = 137 1/2 Marc I Total: 137 1/2 Marc;   [NR 1, 234. 1518]
j^c xiij woger rottz ... vogen for xxiiij ß; Summa ij–^{c} xx– Marc [169 1/2 Marc]
  [113 wog flounder @ 24 ß= 2,712 ß ÷ 16= 169 1/2 Marc I Total: 169 1/2 Marc;   [NR 1, 235. 1518]
j^c xx sidher kwbbespek..hwer side for xxiiij ß [summa ij–^{c} xxx Marc]   [NR 1, 231. 1518]
  [100÷20 sides of blubber, each side 24 ß= 2,880 ß ÷ 16 = 180 Marc I Total: 180 Marc]
ij–^{c} xlviij alne lerrit, huer alne for ij ß. Er Suma xx,v march iiij ß   [NR 1, 433. Akershus, 1531]
  [150 + 48 ell cloth, each ell 2 ß = 396 ß I Total 24 1/2 Marc 4 ß= 396 ß]

It is possible, however, to find relics of "long hundred" terminology in the Norwegina account-books, since decimal hundreds were sometimes called "smalt hundrit" and "long hundred" "store hundrid" for counting animals, fish, and iron:

  ij^{c}xl faarekroppe smalt hwndrit [240 sheep, narrow hundred]   [NR 1, 49, Bergen, 1516-17].
  j^c xxiiij sider swyne fflesk smalt hwndrit [124 sides of pork, narrow hundred]
  iij^c xij t^z danst maltt smale hwndrit [312 barrels Danish malt, narrow hundred]
  iij–^{c} xxx,v farkroppe [2 1/2 hundred + 34 sheep steaks]   [NR 1, 213. 1518]
  j stortt hundritt bergerfiisk er iij– woger regnit for iij– daler   [NR 4, p. 640, 1569-70]

It is however known that around 1300 in some districts, three hundred fish (which was 360) was worth one "mork brend", and that one hundred iron, which was 120 iron sticks, was worth one "kyrlag" or one forngild mork.[17]

Danish accountants seem not to have used "long hundreds" either. The customs in Helsingør, the "Sundtold," used decimal calculation exclusively. "Long hundreds" were not used as an arithmetic system in Denmark, but was used for counting objects such as fish, and planks. Even though I have not found examples of "long hundred" calculation in Denmark and Norway, an 18th century trade manual *Hamburger kontorist*, claims that a long thousand and hundred was used in Copenhagen, as well as in Hamburg, Lübeck, Danzig, and Riga:[18]

  1 groß Tausend hat 10 große Hundert, 60 Snese, oder Steige, oder 1200 Stücke
  1 klein Tausend hat 10 kleine Hundert, 50 Snese, oder Steige, oder 1000 Stücke
  1 groß Hundert hat 2 Schock, 6 Steige, oder 120 Stücke.
  1 klein Hundert aber 5 Steige, oder 100 Stücke

It seems that the Danish-Norwegian state administration some time in the early modern period had introduced a system of accounting that differed from traditional popular methods of calculation. The new accounting methods probably came from the Netherlands or Germany.

The most persistent use of "long hundred" calculation as a distinctive method of calculation is found in Scotland and Iceland . Julian Goodare has shown that the exchequer of Scotland used "long hundred" for calculation of objects in the period 1200-1650, and I have discovered a consistent use of "long hundreds" in the *Treasurer Accounts* in around two hundred examples from the period 1500 to 1570.[19]

The earliest firm evidence of the "long hundred" in Scotland is from 1260:

Summa... xi^c anguille. Expense in servicio regis, vii^{cc} anguille. Item in servicio regine, ix^{xx} anguille. Summa expense, vi^c et lx anguille. Et debet cc et lx anguillas.[20] [Total: 11 hundred eels. Expense to the king, 7 hundred eels, to the queen 9 score eels; Expense, 8 hundred and sixty eels; owe: 2 hundred 60 eels.] - which is in decimal calculation:
[C= 120] Total: 1,320 eels I Expense: 840 to king, 180 to queen= 1,020 + owe= 240+60= 1,320 eels

In an example from 1266 the clerk explained the meaning of *three hundred cows*:

ccc carcosiis vaccarum, - as - xviii^{xx} [18 score= 360 cows][21]

In the 14th century the "long hundred" was used for fish, oatmeal, sheep, stones of wax, pounds of dates, and ells of linen cloth, and in the 15th we can add building stone, and chalders of salt, professor Goodare explains.

In 1330 the "long thousand" was explained for the first time:

"et de viii^m...quorum quodlibet mille faciunt per centenam, sexies viginti, mille et ducentos pisces. Et sic est numerus ix^m vi^c. [Eight thousand, of which each thousand, with a hundred of six score, has one thousand two hundred fish... the [decimal] number is 9,600].

In the *Treasury Accounts*, "hundred" or "C" almost invariably meant "120," except in the monetary sums, where hundred always meant "100." It should, however, be remembered that since a monetary pound had 240 pence, it already incorporated a "long hundred" element; a "long hundred" pounds would therefore imply a duodecimal construction, which was avoided in "long hundred" accounting. The Scottish monetary system was similar to the English: 1 £ was 20 shilling, each of 12 pence.

Only in a rare instances is the word "hundred" or "C" defined as six score, as an indication that the number was a "long hundred". In two examples the cloth and plank measure is defined as six score. In other examples, the superscript Roman numeral ^{xx} indicates the "score." In the following examples vi^{xx} to "j^c" - "C" was the "long hundred" [120], whereas in v^{xx} to "j^c" - "C" was a "short hundred" [100]. However the latter form seems to be an irregular construction for counting items that amounted to a single hundred only.

j^c elne carsay [cloth] reknand sex score for the hundir; ilk elne v s.; summa xxx £
[C=120] 120 x 5 s= 600 s= 30 £   [*TA* II, 42. 1501]

j^c lxxix plankis of aik for the schip, sex score for the hundreth, ilk pece iiij s. ii d.;
        summa xlj £ ix s ij d [*TA* II, 286. 1504].
[C=120] 120 + 79= 199 x 50 d= 9,950 d   I Total: 41 £ x 240= 9,840+108÷2= 9,950 d

Very rarely the normal hundred was used, but it seems only to relate to amounts of a single hundred, and then hundred could be "120" and "100" in the same calculation:

jeistis j^{c}lx, comptand v^{xx} to the j^c, vj^c sparris, comptand vj^{xx} to the j^c
[jests, 100+60, counted 5 score to a hundred, six-hundred, counted 5 score to a hundred;*TA* X 429.1558]

Almost any item would be counted in "long hundreds." That hundred was calculated as "120" can easily be exemplified by the price paid for planks from eastern Germany. Even though they had paid duty in decimal hundreds to the Danish Sund Customs, they were sold by the "long hundred" in Scotland. Since "long hundreds" were used in connection with foreign trade, we must assume that both Scottish buyers and foreign merchants were familiar with this method of calculation:

xiij^c Estland burdis; ilk burd xx d; summa j^c xxx £ [13-hundred boards@ 20 pence;total £130]
[C=120] 13 x 120 x 20 d= 31,200 d= £130 I Total £130   [*TA* III, 84. 1505]
[If C had been 100, the calculation would be incorrect: 1300 £ x 20= 26,000 d= £108 6 s 8 d]

j^c ruf spar, ilk pece ij s j d.; summa xij £ x s [120 spar @ 2 s j d; Total £12 10 s] [*TA* II, 272. 1503]
[C=120] 120 x 25 d= 3,000 d I Total: 12 x 240= 2,880 d + 120 d= 3,000 d

ane hundred four jestis; ilk pece xj s; summa lxviij £ iiij s.   [124 jests @ 11 s; Total: £68 4 s.]
[C= 120] 124 x 11 s= 1,364 s   I Total: 68 £ x 20= 1,360 + 4 s= 1,364 s.   [*TA* III, 85. 1505]

j^c xxx elne blew claith to be gownis to xxx pur men, ilk elne ii s vj d..., summa xviij £ xv s.
[150 ell blue cloth for gowns to 30 poor men, @ 2s 6 d; Total £18 15s.]
[C= 120] (120+30) x 30 d= 4,500 d   I Total: 18 £= 4,320 d +15 s [= 180 d]= 4,500 d [*TA* II, 78. 1502]

ii^c Estland burdis [boards].., ilk burd xvj d.; summa xvi £   [*TA* II, 88. 1501]
[C= 120] 240 x 16= 3,840 d   I Total: 16 £ x 240= 3,840 d

iij^c rauchteris ; ilk pece viij d.; summa xij £   [360 rafter @ 8 d; Total £12]
[C= 120] 360 x 8 d= 2,880 d   I Total: 12 £ x 240= 2,880 d   [*TA* IV, 48. 1508]

The following example shows that hundred was larger than 5 score [5 x 20]:

ij^c five score viij Suethin burdis, ilk pece xij d.; summa xvij £ viij s.
[240 + 100 + 8 Swedish boards@12 d= £17 8s]   [*TA* II, p. 273. 1503]
[C= 120] 240+100+8= 348 x12= 4,176 d ǀ Total: 17 £ x 240= 4,080 + 96= 4,176 d]

It should also be noted that "five hundred" was expressed as "v^c" and not as the Roman numeral "D," but to us the meaning was 600 in "short hundreds" [5x120= 600]. It is therefore obvious that Roman numerals in fact expressed the "long hundred" and not the Roman number concept, and that the "long hundred" dominated over "short hundred" when they counted objects.

v^c sparris [rafter] ..., ilk pece xvij d; summa xlij £ x s.   [5 x 120 rafter @ 17 d = £42 10 s]
[C= 120] 600 x 17 d= 10,200 d ǀ Total: 42 £ x 240= 10,080 + 120= 10,200 d   [*TA* II, 272. 1503]

Finally it should be noted that to the "long hundred" corresponded a "long thousand" of 1,200 items. The "long thousand" is particularly interesting, because it indicates that the "long hundred" was thought to be a system of exponentiation, even though, base ten was "exponentiated" by base twelve.

ane thousand Estland burdis...; ilk burd xix d. summa lxxxxv £ [1200 planks@ 19 d= £95]
[M= 1200] 1,200 x 19= 22,800 d ǀ Total: 95 £ x 240= 22,800 d.   [*TA* III, 88. 1506]

j^{m}v^c lxxxiiij rachteris; ilk pece vij d.; summa liiij £ xix s. [1200+600+84 rafters @ 7 d= £54 19 s]
[M= 1200; C= 120]1200+600+84=1,884 x 7= 13,188 d ǀ Total: 54 £ + 19 s= 13,188 d [*TA* III, 88. 1506]

"Long hundreds" and "long thousands" were used together for example for calculating malt for beer. The example shows that calculation of gallons of beer is only correct when one thousand [m] was 1,200, and one hundred [C] was 120, which is also shown by informing that five score [100] was different from one "hundred." It is very easy to find mistakes in these complicated calculations. In the following example, the result should have been 142 1/2 boll of beer, but the treasurer only calculated 132 1/2 boll, an error of 10 boll or 160 gallons of beer. It is obvious that the methods of accounting were inadequate for handling the growth of the Scottish state economy.

j^{m}j^c fvve skor v gallonis aill, X gallonis for ilk boll [measure] of malt ansuerand to viij chalderis xiiij boll malt, price of ilk boll xiiij s. iiij d; Summa lxxxiiij £ xiijs iiijd
[1200 + 120 + 100 + 5 gallons of beer, 10 gallons@ boll equals 8 chalders, (barrel) + 14 boll; price per boll, 14 s 4 d; Total £94 13 s 4 d.]^{22}
The total was: 1,425 gallons of beer= 142 1/2 boll [10 gallons per boll; 16 *boll*= 1 *calder*].
8 chalder [= 8x16] =128+14 boll= 142[1/2] boll;
The total cost was erroneously calculated for 132 1/2 boll= 22,790 d, not for 142 1/2 boll [24,510 d]

The "Small hundred" was used by the Scots in trade with Dutch or French merchants. Wool fells were sold by v^{xx} to the hundred in the Netherlands, whereas in the domestic trade the hundred was v j^{xx} by definition.^{23} It also seems that the "small hundred" was also used for counting men.

bocht fra Benedict, Duchman, ane thousand sex hundreth lxxj [error for: xxxj]burdis...ilk pece ijs; summa jclxiij £ ij s. [1,000+ 600+ 31 boards @ 2 s; Total £163 2 s]
[M=1000, C= 100] 1,631 x 2 s= 3,262 s ǀ Total: 163 £[= 3260 s]+ 2 s= 3,262 s.   [*TA* II, 83. 1501]

The French ship builder did not calculate in "long hundreds":

Robert Bertoun for iij^{m}iij^{c}xvij fut of gret plankis for the schip, in ilk hundreth five score fut, and for ilk hundreth fut 1 s.; summa lxxxij £ xviij s vj d [3,317 feet planks; @ 1 s/100= £82 18 s 6d]
[M= 1000, C= 100] 3,317 x 6 d= 19,902 d ǀ Total: £82 [=19,680]+216+6= 19,902 d [*TA* II, 285. 1504]

to iii^c and ix futmen in his band ...iii^c and ix £ [309 men were paid 309 £]   [*TA* V, 155. 1517]
to an hundreth and ij futmen in his band j^c ij £ [102 men is paid 102 £]   [*TA* V, 155. 1517]

The "long hundred" came under a cultural pressure from the "small hundred," which was the dominant method of calculation among French and Dutch merchants and craftsmen. The "long hundred" began to disappear gradually from the treasurer's records with the introduction of Arabic numerals, written calculations, and bills; during the sixteenth century, the treasurer began to receive bills from craftsmen, who were paid according to their "accompt."

The Icelandic "long hundred" found in the sagas unlike the Scottish, was not based on a count by the score and even though it is different from the Scottish examples, they seem to be closely related. More examples needs to be found from outside the saga literature than those published by O.S. Reuter in order to show how the "long hundred" was practiced.[24]

It is obvious that the "long hundred" was used in *Rimtól* to inform that there was the 365 days of the year as: *three hundred and five* days; The *Speculum Regis* explains that in book-language all hundreds were decimal, and accordingly the correct number was "iij^c tiræd ok LX dagar."[25] *Rimtól* also explains the 532 years of the Easter cycle as *Fiogur hundrut vetra tolf ræd ok XL oc XII vetur* (480+40+12).[26] The same number is also explained as *fimmhundroth oc XXX oc II ór,* and as *tveir hins fiorda tigar ens setta hundrads tiréds.*[27]

Most occurrences of the word hundred cannot be defined as "long" or "short," and therefore scholars have assumed that the word *hundrad* should always be interpreted as the "long hundred," which is obviously not the case. The table below show the occurrences of "hundred" where the value can be identified found is based on the excerpts for the *dictionary* to be published by the Arnemagnæanske Institute, University of Copenhagen:

LONG HUNDREDS (120) AND NARROW HUNDREDS (100) IN ICELANDIC SAGAS ACCORDING TO THE DICTIONARY OF THE ARNEMAGNÆANSKE COMMISSION. (Thorn=th)

ONE HUNDRED	TWO HUNDRED	THREE HUNDRED	FOUR HUNDRED	FIVE HUNDRED
120 (1x120)	240 (2x120)	360 (3x120)	480 (4x120)	600 (5x120)
halfur tolfta taugur (Rim 2,95) hundrað tolfrøtt (Rim 2,77) tolfreott hundrath (Flat.I 271-16.DN I,344-12)	halft annat hundrað (Rim. 2,95+156) tvav hundrð tolf-ræði (xiiR) CC tolfræð (xii nød) (Jb 162-12.OH 4-21)	thriu hundrut tolfredh (Rim.I,9,II,76-5,139) CCC tolfreoth (Rim 1,12) CCC daga/natta in the year (Rim 1,65+9)	fiogur hundrut tolf raed (Rim 2,157) CCCC tolfraed (Rim 2,173) firda hundraths (Ari iv,I,6)	halft fimta hundrad (Dipl.Isl. 2,438;FlatI,6) halft V hundrad (Dipl.Isl. 2,446) fimm hundraða (Kormak, L.v.7)
100 (1x100)	200 (2x100)	300 (3x100)	400 (4x100)	500 (5x100)
hundrað tirætt (Sk.III,43 Fl.VIII,33) C.tiræett ok XX. (Stj. 55) v hins tiunda tigher (Rim. 1,34) tiu tigu= C tiræett (Kristnisaga)	tvö hundrað tiræd sem C tvö I látinu (Grammar) halft annat hundrad (Hauksbok 151) II hundrud ok XL (SnE(W)21-12).	thridia hundrads (Rim 1,64) CCC daga tiredum (Rim 1,61)		finm hundrod oc xxx oc ii (Rim.1,62) fimm hundrut tiræd 32 ar (Rim.II,137-18) finm hundrud+firum (Grm 23+24)
SIX HUNDRED	SEVEN HUNDRED	EIGHT HUNDRED	NINE HUNDRED	TEN/TWELVE HDR.
720 (6x120)	840 (7x120)	960 (8x120)	1080 (9x120)	1200 (10x120)
stunder i manadi 600 (=720) (Rim 128-17)	DCC tolfraed (thattr THorv.)	atta hundraða tolf-ræd (DNII,79-36)	halft niunda hundrad (Dipl.Isl.1,204)	*thusund (Sverre S.Fm 8,448) * x/tio hundrud (StuK 213-18 a.o.)
600 (6x100)	700 (7x100)	800 (8x100)	900 (9x100)	1000
setta hundrad tireds (Rimb.61-14) setta hundrads ti-roeds (Rim 1,32)	*siau hundrod manna (Gudr. III,7)	viii hundrut tired (Alex.112-31) *atta hundrud (Grm 23)	hundrud niu (Hym. 8)	tolf hundrud (HHu. 1,25) xii hundredum tyrae-dum (Rymb.p.404)

Another method to reveal that a hundred was a "long hundred" is to observe the occurrence of 110 in combination with the word hundred, which would then prove that the "hundred" used was larger than eleven-tens. An expression for 110 occurs a few times, and there are two examples of 110 in combination with hundred:[28]

Ellifuu tigher - Ellefutigi, - ælliufu tighi - ællifutigi

Eleventh ten, "eleventy" [DI: III, 289-4. 1374; IV 117-2. 1397. DN: II, 79-31. 1309; III, 107-23. 1317]

thriu hundrud ellefutigir og sio aar   [DI IV, 259-3. 1417]
[C= 120] 360 + 110 + 7= 477 years

thushundrat thriu hundrut ellefu tiger og atta ar   [DI IV, 266-3. 1418]
[M=1,200, C= 120] 1200 + 360 + 110 + 8= 1,678 years

I have not been able to study the "long hundred" in Icelandic account books, except for the 17th century description of the Icelandic taxation system by Arent Berntsen. Apparently he did not understand the system, but recorded the information in Arabic numerals, which at times appear to be illogical: "*100 Fiske/er... 6 Snese/eller 120 Fiske*" [100=120] "*udi 100 er 6 snese*" "*1 Stort 100 ... er 6 Snese*". [29]

Berntsen's explanation of the Icelandic system of land taxation as it is printed makes no sense, though when converting the information into "long hundreds," most irregularities disappear. It should be noticed that the score is used in this text.

Ißlands Iordegodsis Taxt, eller Pris efter Landskylds høvde (udi 100, er 6 Snese).

Frijt	Frijt	Fiske [120]	Fiske [100]	Hundret Jord [120]	H. J.	[100]	Rdlr[120]	R[100]
600	720 #	1200~	1440 #	100 Hundret~	100[=120]H>	14400	960>	1152
500	600	1000	1200	5 snes hund	100H>	12000	800>	960
400	480 *	800	960 *	4 snes hund	80H	9600 *	644 {640}	768
360 ~	420	700	840	3 1/2 snes hund	70H	8400	560	672
300	360 #	600	720 #	3 snes hund	60H	7200	476 {480}	576
260 ~	300	500	600	2 1/2 snes hund	50H>	6000	400 >	480
200	240 *	400	480 *	40 Hundret		4800 *		
160 ~	180 #	300	360 #	30 Hundret		3600 #		
100 ~	120 *	200 ~	240 *	20 Hundret		2400 *		
80 ~	80 #	140	160 # [2 1/4]	13 Hundret oc 40 Alen J		1600 # [2 1/4]		
70 Al	70	120~	140	11 1/2 Hundret oc 20 Alen J		1400		
60	60*	100~	120 *	10 Hundret		1200 *		
50~Al	50	5 Snes [100]	100	800 oc 40 Alne Jord		1000		

It seems that all numbers in the table larger than 80 were "long hundreds;" Berntsen explains in the heading that 100 was 6 score [=120]. The table must be converted into normal hundreds to make it understandable to us. The original text is boldfaced, and I have noted numbers with " ~ " irregularities if one would perceive the numbers as decimal; " { }" indicates my suggestion to correction of errors in the conversion to Rigsdaler; the signs *, > and # mark connections between numbers, which are multiples of each other.

The land tax seem to have been an 8% tax on the value of land [hundret Jord], and the sum was stated in "long hundreds" of Rigsdaler, which Berntsen listed for the six largest classes. Two minor errors seems to occur in the sums of Rigsdaler, 644 Rigsdaler, which should have been "long hundred" 640 Rigsdaler or "short hundred" 768 Rigsdaler. The "long" 476 Rigsdaler, should have been "long" 480 Rigsdaler, which would be "short" 576 Rigsdaler [7200 x 8%= 576].

The table clearly shows how difficult it was to calculate long hundreds in Arabic numerals on paper, whereas the calculations was done fairly easily on a counting table. Julian Goodare describes very well how to calculate "long hundreds" on the counting table in his article.

Berntsen's example of how fish were counted in Iceland exemplifies how counting was performed in "long hundreds" and "long thousands." One load of fish contains "1000" fish, but since hundred equals six score or 120 fish (which is also three væt fish, or 3 x 40). Since "100" meant 120, "1000" meant 1,200.[30]

It is also possible to find the "long hundred" in England, but it seems to have disintegrated as a system at an early time. A fifteenth century manuscript from London[31], shows that the word "hundred," or the Roman numeral "C", had several numerical meanings. Even though there are many different examples of hundreds in the manuscript, two major versions of a hundred can be distinguished, a five-score hundred and a six-score hundred:

in linear measures the Roman numeral "C" meant v^{xx} (C=five score=100),

for cheese, the gret "C" meant v^{xx} xij pound [hundredweight] (C=five score + 12=112),

when counting herring "C" meant vj^{xx} iiij (C=six score + four=124).

for counting fells "C" meant vj^{xx} (C=six score=120).

The text explains that fells could be counted in two ways, by counting six score to the hundred, or ten dozen as a hundred. (vi^{xx} fells go to the C, and X dozen make a C).

The use of a "long hundred" of 124 in counting of fish can be verified from a fisherman's tallies from the Isle of Man, dated 1888. The label tells that the small notches had the nominal value 100, but the real value was 124 fish; the larger nominal "500" meant 620 fish, was called a mease. As late as in 1886 a mease of herring in Scotland consisted of five hundred - in which the hundred was "120."[32]

In conclusion it should be stated that the existence of "long hundred" calculation is easily documented in Scotland and Iceland, and to some extent in England, whereas very few examples can be found in the rest of Scandinavia and in the Germanic area. It therefore seems unlikely that the "long hundred" and counting by the score was of Scandinavian origin, as it has often been suggested. Rather, the "long hundred" and counting by the "score" seem to have evolved among the Celts in the British Isles and probably it spread to Iceland. The examples are still too few, however, to establish the position of the long hundred in Germanic and in Celtic culture with certainty. "Long hundreds" may have emerged independently, or from contact with Roman methods of calculation and measuring.

The "long hundred" could be calculated on tally stick, and was a very flexible system of calculation compared to other contemporary arithmetic methods. The many divisors of the higher base 120, which combined the divisors of 10, 12, and 20 made calculation easy for practical purposes. The ancient "long hundred" number concept differed substantially from the modern one by having a "spatial" number concept. The "long hundred" was not an exceptional method of calculation; counting by 60 or 120 was widely used in many civilizations.

"Long hundred" arithmetic gradually disappeared with the introduction of Arabic numerals and written calculation. The last trace of the "long hundred" disappeared when British coinage became decimal in the 1970s. Decimal calculation was introduced long after the middle ages. The introduction of exponentiation of base ten, ten numerical symbols, the positional calculation, and decimal fractions, decimal calculation revolutionized modern arithmetic on which our scientific culture is based. The victory of decimal calculation is so complete that it is almost impossible for us to understand how other methods of calculation could have been used in earlier times.

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8 Otto Sigfrid Reuter, "Zur Bedeutungsgeschichte des hundrað im Altwestnordischen" in *Arkiv för Nordisk Filologi* (Lund, 1933) p. 36-67.
Ibid, "Urnordischer und eurasischer Zählbrauch" in *Mannus*, Bd 25 (Leipzig, 1933) p. 353-383.

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14 E.g., Holger Pedersen, *Vergleichende Grammatik der keltischen Sprachen.* (Göttingen, 1913), p. 130ff. - *Oxford English Dictionary*, and other dictionaries, vide hundred.

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20 Goodare, *op. cit.* p. 400, reference: J. Stuart, *Exchequer Rolls of Scotland* i (Edinburgh, 1878-1904), p. 7. Accounts of the keeper of the bishopric of Glasgow. [ER]

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Ibid, "Urnordischer und eurasischer Zählbrauch" *Mannus*, Bd 25 (Leipzig, 1933) p. 353-383.

26 Alfrædi, *Rím, II*, 157-12. *op cit* p. 530..

27 *Rím* 1, 62, and 1, 32.

28 I am grateful to the Arnemagnæanske Institute for giving me access to the index cards to the *Dictionary* for the unpublished entries "tigr," "tiu-tigir," "ti-ræðr," "tolfræðr," "hundrað."

29 Arent Berntsen: *Danmarckis oc Norgis Fructbar Herlighed* (Copenhagen, 1656, Repr. 1971), p. 530, 328, 552.

30 Berntsen, *op cit.* p. 530.

31 Balliol College, Oxford, Manuscript 354, and transcription by D. C. Browning, p. 15-16, 367.

32 Oxford English Dictionary 2. ed. (Oxford, 1989) Vol. 7: Hundred § 3, p. 491.